# PROBABILISTIC METHODS IN PDE

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**PREREQUISITES :** Appropriate for students with MSc in Pure Mathematics with specialization in Analysis and/or Probability Theory. Prerequisite courses: Measure Theory, Functional Analysis, Probability Theory, Stochastic Processes.

**INTENDED AUDIENCE :** Doctoral students or researchers in the area of partial differential equations or stochastic processes who wish to learn the probabilistic techniques in PDE for solving research problems either in pure or applied mathematics such as Mathematical Finance or Mathematical Physics.

**INDUSTRY SUPPORT :** This course develops tools to solve deterministic Evolution Problem arising in physical scenarios with random noise. The evolution problems include, for example, heat equation and option price equations. Therefore, the cutting edge R&D sectors of Finance industry should value this course.

**COURSE OUTLINE :**Probabilistic method in PDE is equally used in Pure and Applied Mathematics research. This is regarded as a very powerful tool by the researchers working on the theory of differential equations. However, as the topic demands expertise on both PDE and probability theory, an initiative to teach the topic as a structured course is vastly absent globally, including in India. There is hardly any lecture note or a course accessible for the mathematics students. There is no book for mathematics students which focuses on this topic and assembles all important aspects suitable for an introduction to this topic. Young researchers like PhD students or junior postdoctoral fellows, who aspire to learn the topic, resorts on several different books and study by themselves, which often consumes considerable amount of their productive time.

To change the present discouraging scenario and to boost up research on this very powerful and vibrant topic, I have designed this introductory course. I have offered this once informally at IISER Pune and then officially at Justus-Liebig University, Giessen, Germany for research students. This course, although an advanced one, attracts students with a background of PDE, Probability Theory, Mathematical Finance, or Mathematical Physics. This course allows a researcher to confidently take up an original research problem in the related field.

This course content is mainly based on two different books, one on stochastic calculus and another on semigroup theory. Many theorems would be proved in the lectures with greater details than the reference books.

**ABOUT INSTRUCTOR :**

Prof. Anindya Goswami received his Bachelor's degree in Mathematics from St. Xavier's College, Calcutta in 2002. Later in the same year, he joined the Integrated Ph.D. program in the Department of Mathematics in Indian Institute of Science, Bangalore. Following the completion of MS degree in 2005, he received the SPM fellowship as part of the National Award for best performance in National Eligibility Test in Mathematical Sciences. He was bestowed with the Doctorate degree from the Department of Mathematics, IISc in the year 2008. The following three years, he carried out postdoctoral research in the University of Twente, Netherlands; INRIA- Rennes, France; and Technion- Israel Institute of Technology, Israel respectively. He joined IISER Pune as an Assistant Professor in fall, 2011. Since then, he has offered a variety of graduate and undergraduate courses- Multivariable Calculus, Point-set Topology, Measure Theory, Functional Analysis, Numerical Analysis, Measure Theoretic Probability Theory, Stochastic Processes, Mathematical Finance, to name a few. He was reappointed at the same department as an Associate Professor in spring, 2018. His current research interest comprises of Non-cooperative Stochastic Dynamic Game, Stochastic Control, Mathematical Finance, and Queuing Network.

**COURSE PLAN :**

**Week 1 :** Mathematical formulation of stochastic processes

**Week 2 :** Brief review of L2 theory of stochastic integration

**Week 3 :** Ito’s formula

**Week 4 :** Probabilistic method in Dirichlet problem

**Week 5 :** Further topics of Dirichlet problem and Probabilistic method in heat equation

**Week 6 :** Further topics of Probabilistic method in heat equation

**Week 7 :** Feynman Kac formula

**Week 8 :** Stochastic differential equations

**Week 9 :** PDE with general elliptic operators

**Week 10 :** Feynman Kac formula and its abstraction with semigroup theory

**Week 11 :** Mild solution to linear evolution problems

**Week 12 :** Mild solution to semilinear evolution problem