LINEAR ALGEBRA

PROF. DILIP P. PATIL
Department of Mathematics
IISc Bangalore

INTENDED AUDIENCE : BSc / BE /ME/ MSc / PhD
INDUSTRIES APPLICABLE TO : Language of Set Theory; Elementary Algebra and Calculus

COURSE OUTLINE :
The main purpose of this course in the study of linear operators on finite dimensional vector spaces. The idea is to emphasize the simple geometric notions common to many parts of mathematics and its applications. Except for an occasional reference to undergraduate mathematics, the course will be self-contained. The algebraic co-ordinate free methods will be adopted through out the course. These methods are elegant and as elementary as the classical as coordinatized treatment. The scalar field will be arbitrary (even a finite field), however, in the treatment of vector spaces with inner products, special attention will be given to the real and complex cases. Determinants via the theory of multilinear forms. Variety of examples of the important concepts. The exercises will constitute significant asstion ; ranging from routine applications to ones which will extend the very best students.

ABOUT INSTRUCTOR :
Prof. Dilip P. Patil received B. Sc. and M. Sc. in Mathematics from the University of Pune in 1976 and 1978, respectively. From 1979 till 1992 he studied Mathematics at School of Mathematics, Tata Institute of Fundamental Research, Bombay and received Ph. D. through University of Bombay in 1989. Currently he is a Professor of Mathematics at the Departments of Mathematics, Indian Institute of Science, Bangalore. At present he is a Visiting Professor at the Department of Mathematics, IIT Bombay. He has been a Visiting Professor at Ruhr-Universitt Bochum, Universitt Leipzig, Germany and several universities in Europe and Canada. His research interests are mainly in Commutative Algebra and Algebraic Geometry.

COURSE PLAN :
Week 1
Introduction to Algebraic Structures - Rings and Fields.
Definition of Vector Spaces.
Examples of Vector Spaces.
Definition of subspaces.
Examples of subspaces.
Week 2
Examples of subspaces (continued).
Sum of subspaces.
System of linear equations.
Gauss elimination.
Generating system , linear independence and bases.
Week 3
Examples of a basis of a vector space.
Review of univariate polynomials.
Examples of univariate polynomials and rational functions.
More examples of a basis of vector spaces.
Vector spaces with finite generating system.
Week 4
Steinitzs exchange theorem and examples.
Examples of finite dimensional vector spaces.
Dimension formula and its examples.
Existence of a basis.
Existence of a basis (continued).

Week 5
Existence of a basis (continued).
Introduction to Linear Maps.
Examples of Linear Maps.
Linear Maps and Bases.
Pigeonhole principle in Linear Algebra.

Week 6
Interpolation and the rank theorem.
Examples.
Direct sums of vector spaces.
Projections.
Direct sum decomposition of a vector space.

Week 7
Dimension equality and examples.
Dual spaces.
Dual spaces (continued).
Quotient spaces.
Homomorphism theorem of vector spaces.

Week 8
Isomorphism theorem of vector spaces.
Matrix of a linear map.
Matrix of a linear map (continued).
Matrix of a linear map (continued).
Change of bases.

Week 9
Computational rules for matrices.
Rank of a matrix.
Computation of the rank of a matrix.
Elementary matrices.
Elementary operations on matrices.

Week 10
LR decomposition.
Elementary Divisor Theorem.
Permutation groups.
Canonical cycle decomposition of permutations.
Signature of a permutation.

Week 11
Introduction to multilinear maps.
Multilinear maps (continued).
Introduction to determinants.
Determinants (continued).
Computational rules for determinants.

Week 12
Properties of determinants and adjoint of a matrix.
Adjoint-determinant theorem.
The determinant of a linear operator.
Determinants and Volumes.
Determinants and Volumes (continued).