Selected Topics in Mathematical Physics - Video course

COURSE OUTLINE

Analytic functions of a complex variable. Calculus of residues, Linear response; dispersion relations. Analytic continuation and the gamma function. Mobius transformations. Multivalued functions; integral representations. Laplace transforms. Fourier transforms. Fundamental Green function for the Laplacian operator. The diffusion equation. Green function for the Helmholtz operator; nonrelativistic scattering. The wave equation. The rotation group and all that.

COURSE DETAIL

Modules	lectures	Topics
Module 1.	Lecture 1.	Analytic functions of a complex variable (Part I): Complex numbers. Equations to curves in the plane in terms of z and z*. The Riemann sphere and stereographic projection. Analytic functions of z and the Cauchy-Riemann conditions. The real and imaginary parts of an analytic function.
	Lecture 2.	Analytic functions of a complex variable (Part II): The derivative of an analytic function. Power series as analytic functions. Convergence of power series.
Module 2.	Lecture 3.	Calculus of residues (Part I): Cauchy's integral theorem. Singularities--- removable singularity, simple pole, multiple pole, essential singularity. Laurent series. Singularity at infinity. Accumulation point of poles. Meromorphic functions. Cauchy's integral formula.
	Lecture 4.	Calculus of residues (Part II): Solution of difference equations using generating functions and contour integration.
	Lecture 5.	Calculus of residues (Part III): Summation of series using contour integration. Evaluation of definite integrals using contour integration.

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Physics

Pre-requisites:

- A basic course in mathematical methods used in physics

Coordinators:

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	Lecture 6.	Calculus of residues (Part IV): Contour integration. Mittag-Leffler expansions of meromorphic functions.	
Module 3.	Lecture 7.	Linear response; dispersion relations (Part I): Causal, linear, retarded response. Dynamic susceptibility. Symmetry properties of the dynamic susceptibility. Dispersion relations. Hilbert transform pairs.	
	Lecture 8.	Linear response; dispersion relations (Part II): Subtracted dispersion relations. Admittance of an LCR circuit. Discrete and continuous relaxation spectra.	
Module 4.	Lecture 9.	Analytic continuation and the gamma function (Part I): Definition of the gamma function and its analytic continuation. Analytic properties.	
	Lecture 10.	Analytic continuation and the gamma function (Part II): Connection with gaussian integrals. Mittag-Leffler expansion of the gamma function. Logarithmic derivative of the gamma function. Infinite product representation for the gamma function. The beta function. Reflection and duplication formulas for the gamma function.	
Module 5.	Lecture 11.	Mobius transformations (Part I): Conformal mapping. Definition of a Mobius transformation and its basic properties.	
	Lecture 12.	Mobius transformations (Part II): Fixed points of a Mobius transformation. The cross-ratio. Normal form of a Mobius transformation. Iterates of a Mobius transformation.	
	Lecture 13.	Mobius transformations (Part III): Classification of Mobius transformations. The isometric circle. Group properties; the Mobius group. The Mobius group over the reals. The modular group. The invariance group of the unit circle. Connection with the pseudo-unitary group SU(1,1). The group of cross-ratios.	
	Lecture 14.	Multivalued functions; integral representations (Part I): Branch points and branch cuts. Algebraic and logarithmic branch points, winding point. Riemann sheets.	
		Multivalued functions; integral representations (Part	

Module 6.	Lecture 15.	II): Contour integrals in the presence of branch points. An integral involving a class of rational functions. Contour integral representation for the gamma function.
	Lecture 16.	Multivalued functions; integral representations (Part III): Contour integral representations for the beta function and the Riemann zeta function. Connection with Bernoulli numbers. Zeroes of the zeta function. Statement of the Riemann hypothesis.
	Lecture 17.	Multivalued functions; integral representations (Part IV): Contour integral representations of the Legendre functions of the first and second kinds. Singularities o f functions defined by integrals. End-point and pinch singularities, examples. Singularities of the Legendre functions. Dispersion relations for the Legendre functions.
Module 7.	Lecture 18.	Laplace transforms (Part I): Definition of the Laplace transform. The convolution theorem. Laplace transforms of derivatives. The inverse transform, Mellin's formula. The LCR series circuit. Laplace transform of the Bessel and modified Bessel functions of the first kind. Laplace transforms and random processes: the Poisson process.
	Lecture 19.	Laplace transforms (Part II): Laplace transforms and random processes: biased random walk on a linear lattice and on a d-dimensional lattice.
Module 8.	Lecture 20.	Fourier transforms (Part I): Fourier integrals. Parseval's formula for Fourier transforms. Fourier transform of the delta function. Relative `spreads' of a Fourier transform pair. The convolution theorem. Generalization of Parseval's formula. Iterates of the Fourier transform operator.
	Lecture 21.	Fourier transforms (Part II): Unitarity of the Fourier transformation. Eigenvalues and eigenfunctions of the Fourier transform operator.
	Lecture 22.	Fourier transforms (Part III): The Fourier transform in d dimensions. The Poisson summation formula. Some illustrative examples. Generalization to higher dimensions.
	Lecture 23.	Fundamental Green function for the laplacian operator (Part I): Green functions. Poisson's equation. The

Module 9.	Lecture 23.	fundamental Green function for the laplacian operator. Solution of Poisson's equation for a spherically symmetric source.	
	Lecture 24.	Fundamental Green function for the laplacian operator (Part II): The Coulomb potential in d dimensions. Ultraspherical coordinates. A divergence problem. Dimensional regularization. Direct derivation using Gauss' Theorem. The Coulomb potential in d = 2 dimensions.	
Module 10.	Lecture 25.	The diffusion equation (Part I): Fick's laws of diffusion. Diffusion in one dimension: Continuum limit of a random walk. The fundamental solution. Moments of the distance travelled in a given time.	
	Lecture 26.	The diffusion equation (Part II): The fundamental solution in d dimensions. Solution for an arbitrary initial distribution. Finite boundaries. Solution by the method of images. Diffusion with drift. The Smoluchowski equation. Sedimentation.	
	Lecture 27.	The diffusion equation (Part III): Reflecting and absorbing boundary conditions. Solution by separation of variables. Survival probability.	
	Lecture 28.	The diffusion equation (Part IV): Capture probability and first-passage-time distribution. Mean first passage time.	
Module 11.	Lecture 29.	Green function for the Helmholtz operator; nonrelativistic scattering (Part I): The Helmholtz operator. Physical application: scattering from a potential in nonrelativistic quantum mechanics. The scattering amplitude; differential cross-section.	
	Lecture 30	Green function for the Helmholtz operator; nonrelativistic scattering (Part II): Total cross-section for scattering. Outgoing wave Green function for the Helmholtz operator. Integral equation for scattering.	
	Lecture 31.	Green function for the Helmholtz operator; nonrelativistic scattering (Part III): Exact formula for the scattering amplitude. Scattering geometry and the momentum transfer. Born series and the Born approximation. The Yukawa and Coulomb potentials. The Rutherford scattering	

		formula.
Module 12.	Lecture 32.	The wave equation (Part I): Formal solution for the causal Green function of the wave operator. The solution in (1+1) and (2+1) dimensions.
	Lecture 33.	The wave equation (Part II): The Green function in (3+1) dimensions. Retarded solution of the wave equation. Remarks on propagation in spatial dimensions d > 3. Differences between even and odd spatial dimensionalities. Energy-momentum relation for a relativistic free particle. The Klein-Gordon equation and the associated Green function.
Module 13.	Lecture 34.	The rotation group and all that (Part I): Rotations of the coordinate axes. Orthogonality of rotation matrices. Proper and improper rotations. Generators of infinitesimal rotations in 3 dimensions. Lie algebra of generators. Rotation generators in 3 dimensions transform like a vector. The general rotation matrix in 3 dimensions.
	Lecture 35.	The rotation group and all that (Part II): The finite rotation formula for a vector. The general form of the elements of U(2) and SU(2). Relation between the groups SO(3) and SU(2).
	Lecture 36.	The rotation group and all that (Part III): The 2-to-1 homomorphism between SU(2) and SO(3). The parameter spaces of SU(2) and SO(3). Double connectivity of SO(3). The universal covering group of a Lie group. The group SO(2) and its covering group. The groups SO(n) and Spin(n). Tensor and spinor representations. Parameter spaces of U(n) and SU(n). A bit about the fundamental group (first homotopy group) of a space. Examples.

References:

The topics covered in this course are ‘classical’, and the number of texts and monographs on them is quite large. What follows is a short suggested list of books for reference and for additional reading.

1. Ablowitz, M. J. and Fokas, A. S., Complex Variables - Introduction and Applications
2. Ahlfohrs, L. V., ComplexAnalysis: An Introduction to the Theory of Analytic Functions of One Complex Variable
3. Conway, J. B., Functions of One Complex Variable
4. Copson, E. T., An Introduction to the Theory of Functions of a Complex Variable
5. Courant, R. and Hilbert, D., Methods of Mathematical Physics, Vols, 1 & 2
6. Dennery, P. and Krzywicki, A., Mathematics for Physicists
7. Ford, L. R., Automorphic Functions
8. Gilmore, R., Lie Groups, Lie Algebras and Some of Their Applications
9. Hille, E., Analytic Function Theory Vols. 1 & 2
10. Korner, T. M., Fourier Analysis

11. Lighthill, M. J., An Introduction to Fourier Analysis and Generalized Functions
12. Nikiforov, A. F. and Uvarov V. B., Special Functions of Mathematical Physics
13. Sansone, G. and Gerretsen, I., Lectures on the Theory of Functions of One Complex Variable, I and II
14. Sneddon, I. N., The Use of Integral Transforms
15. Titchmarsh, E. C., Introduction to the Theory of Fourier Integrals
16. Titchmarsh, E. C., The Theory of Functions
17. Whittaker, E. T. and Watson, G. N., A Course of Modern Analysis
18. Widder, D. V., The Laplace Transform
19. Wu, T.-Y. and Ohmura, T., Quantum Theory of Scattering